

Charge independence breaking and charge symmetry breaking in the nucleon–nucleon interaction from effective field theory

E. Epelbaum,^{a,b#1} Ulf-G. Meißner^{b#2}

^a*Ruhr-Universität Bochum, Institut für Theoretische Physik II
D-44870 Bochum, Germany*

^b*Forschungszentrum Jülich, Institut für Kernphysik (Theorie)
D-52425 Jülich, Germany*

Abstract

We discuss charge symmetry and charge independence breaking in an effective field theory approach for few–nucleon systems. We systematically introduce strong isospin–violating and electromagnetic operators in the theory. The charge dependence observed in the nucleon–nucleon scattering lengths is due to one–pion exchange and one electromagnetic four–nucleon contact term. This gives a parameter free expression for the charge dependence of the corresponding effective ranges, which is in agreement with the rather small and uncertain empirical determinations. We also compare the low energy phase shifts of the nn and the np system.

DEDICATED TO WALTER GLÖCKLE ON THE OCCASION OF HIS 60th BIRTHDAY

^{#1}email: evgeni.epelbaum@hadron.tp2.ruhr-uni-bochum.de

^{#2}email: Ulf-G.Meissner@fz-juelich.de

1. It is well established that the nucleon–nucleon interactions are charge dependent (for a review, see e.g.[1]). For example, in the 1S_0 channel one has for the scattering lengths a and the effective ranges r (n and p refers to the neutron and the proton, in order)

$$\begin{aligned}\Delta a_{\text{CIB}} &= \frac{1}{2}(a_{nn} + a_{pp}) - a_{np} = 5.7 \pm 0.3 \text{ fm} , \\ \Delta r_{\text{CIB}} &= \frac{1}{2}(r_{nn} + r_{pp}) - r_{np} = 0.05 \pm 0.08 \text{ fm} .\end{aligned}\tag{1}$$

These numbers for charge independence breaking (CIB) are based on the Nijmegen potential and the Coulomb effect for pp scattering is subtracted based on standard methods. The charge independence breaking in the scattering lengths is large, of the order of 25%, since $a_{np} = (-23.714 \pm 0.013) \text{ fm}$. In addition, there are charge symmetry breaking (CSB) effects leading to different values for the pp and nn threshold parameters,

$$\begin{aligned}\Delta a_{\text{CSB}} &= a_{pp} - a_{nn} = 1.5 \pm 0.5 \text{ fm} , \\ \Delta r_{\text{CSB}} &= r_{pp} - r_{nn} = 0.10 \pm 0.12 \text{ fm} .\end{aligned}\tag{2}$$

Both the CIB and CSB effects have been studied intensively within potential models of the nucleon–nucleon (NN) interactions. In such approaches, the dominant CIB comes from the charged to neutral pion mass difference in the one–pion exchange (OPE), $\Delta a_{\text{CIB}}^{\text{OPE}} \sim 3.6 \pm 0.2 \text{ fm}$. Additional contributions come from $\gamma\pi$ and 2π (TPE) exchanges. Note also that the charge dependence in the pion–nucleon coupling constants in OPE and TPE almost entirely cancel. In the meson–exchange picture, CSB originates mostly from $\rho - \omega$ mixing, $\Delta a_{\text{CSB}}^{\rho-\omega} \sim 1.2 \pm 0.4 \text{ fm}$. Other contributions due to $\pi - \eta$, $\pi - \eta'$ mixing or the proton–neutron mass difference are known to be much smaller.

Within QCD, CSB and CIB are of course due to the different masses and charges of the up and down quarks. Such isospin violating effects can be systematically analyzed within the framework of chiral effective field theories. In the two–nucleon sector, a complication arises due to the unnaturally large S–wave scattering lengths. This can be dealt with in two ways. One is the “hybrid” approach due to Weinberg [2], in which chiral perturbation theory is applied to the interaction kernel sewed to realistic nuclear wave functions obtained by conventional means. This approach has been successfully applied to a variety of processes which are dominated by OPE, a particularly striking one being the prediction of the electric dipole amplitude for neutral pion production off deuterons [3] recently measured at SAL [4]. A different and more *systematic* fashion to deal with the unnaturally large scattering lengths is the recently proposed power divergence subtraction scheme (PDS) proposed by Kaplan, Savage and Wise (KSW) [5].^{#3} Essentially, one resums the lowest order local four–nucleon contact terms $\sim C_0(N^\dagger N)^2$ (in the S–waves) to generate the large scattering lengths and treats the remaining effects perturbatively, in particular also pion exchange. This means that most low–energy observables are dominated by contact interactions. The chiral expansion for NN scattering entails a new scale Λ_{NN} of the order of 300 MeV, so that one can systematically treat external momenta up to the size of the pion mass. There have been suggestions that the radius of convergence can be somewhat enlarged [6], but in any case Λ_{NN} is considerably smaller than

^{#3}There exist by now modifications of this approach and it has been argued that it is equivalent to cut–off schemes. We do not want to enter this discussion here but rather stick to its original version.

the typical scale of about 1 GeV appearing in the pion–nucleon sector. A status report of the various observables calculated within this framework can be found in ref.[7]. In this context, it appears to be particularly interesting to study CIB (or in general isospin violation) which is believed to be dominated by long range pion effects. That is done here.^{#4} First, we write down the leading strong and electromagnetic four–nucleon contact terms. It is important to note that in contrast to the pion or pion–nucleon sector, one can not easily lump the expansion in small momenta and the electromagnetic coupling into one expansion but rather has to treat them separately. Then we consider in detail CIB. The leading effect starts out at order αQ^{-2} , where Q is the generic expansion parameter in the KSW approach. It stems from OPE plus a contact term of order α with a coefficient of natural size that scales as Q^{-2} . Similarly, the leading CSB effect are four–nucleon contact terms of order α and order $m_u - m_d$, which also scale as Q^{-2} . While in the case of $E_0^{(1)}$ this scaling property is enforced by a cancellation of a divergence, the situation is a priori different for CSB. However, for a consistent counting of all isospin breaking effects related to strong or em insertions, one should count the quark mass difference and virtual photon effects similarly. Note, however, that these CIB and CSB terms are numerically much smaller than the leading strong contributions which scale as Q^{-1} because $\alpha \ll 1$ and $(m_u - m_d)/\Lambda_\chi \ll 1$. The corresponding constants, which we call $E_0^{(1,2)}$, together with the strong parameters (as given in the work of KSW) can be determined by fitting the three scattering lengths a_{pp} , a_{nn} , a_{np} and the np effective range.^{#5} That allows to predict the momentum dependence of the np and the nn 1S_0 phase shifts. Based on these observation, we can in addition give a general classification for the relevant operators contributing to CIB and CSB in this scheme. Additional work related to long–range Coulomb photon exchange is necessary in the proton–proton system. We do not deal with this issue here but refer to recent work using EFT approaches in refs.[11][12].

2. First, we discuss the various parts of the effective Lagrangian underlying the analysis of isospin violation in the two–nucleon system. To include virtual photons in the pion and the pion–nucleon system is by now a standard procedure [13]–[19]. The lowest order (dimension two) pion Lagrangian takes the form

$$\mathcal{L}_{\pi\pi} = \frac{f_\pi^2}{4} \langle \nabla_\mu U \nabla^\mu U^\dagger + \chi U^\dagger + \chi^\dagger U \rangle + C \langle QUQU^\dagger \rangle, \quad (3)$$

with $f_\pi = 92.4$ MeV the pion decay constant, ∇_μ the (pion) covariant derivative containing the virtual photons, $\langle \rangle$ denotes the trace in flavor space, χ contains the light quark masses and the last term, which contains the nucleon charge matrix $Q = e \text{diag}(1, 0)$,^{#6} leads to the charged to neutral pion mass difference, $\delta m^2 = m_{\pi^\pm}^2 - m_{\pi^0}^2$, via $\delta m^2 = 8\pi\alpha C/f_\pi^2$, i.e. $C = 5.9 \cdot 10^{-5} \text{GeV}^4$. Note that to this order the quark mass difference $m_u - m_d$ does not appear in the meson Lagrangian (due to G–parity). That is chiefly the reason why the pion mass difference is almost entirely an electromagnetic (em) effect. The equivalent

^{#4}For a first look at these effects in an EFT framework, see the work of van Kolck [8][9]. Electromagnetic corrections to the one–pion exchange potential have been considered in [10].

^{#5}Whenever we talk of the pp system, we assume that the Coulomb effects have been subtracted.

^{#6}Or equivalently, one can use the quark charge matrix $e(1 + \tau_3)/2$.

pion–nucleon Lagrangian to second order takes the form

$$\begin{aligned} \mathcal{L}_{\pi N}^{\text{str}} = & N^\dagger \left(iD_0 - \frac{g_A}{2} \vec{\sigma} \cdot \vec{u} \right) N + N^\dagger \left\{ \frac{\vec{D}^2}{2M} + c_1 \langle \chi_+ \rangle + \left(c_2 - \frac{g_A^2}{8M} \right) u_0^2 + c_3 u_\mu u^\mu \right. \\ & \left. + \frac{1}{4} \left(c_4 + \frac{1}{4M} \right) [\sigma_i, \sigma_j] u_i u_j + c_5 \left(\chi_+ - \frac{1}{2} \langle \chi_+ \rangle \right) + \dots \right\} N, \end{aligned} \quad (4)$$

which is the standard heavy baryon effective Lagrangian in the rest–frame $v_\mu = (1, 0, 0, 0)$. M is the nucleon mass and u_μ the chiral viel–bein, $u_\mu \sim -i\partial_\mu \phi / f_\pi + \dots$. The four–nucleon interactions to be discussed below do not modify the form of this Lagrangian (for a general discussion, see e.g. ref[20]). Strong isospin breaking is due to the operator $\sim c_5$. Electromagnetic terms to second order are given by [18]

$$\mathcal{L}_{\pi N}^{\text{em}} = f_\pi^2 N^\dagger \left\{ f_1 \langle Q_+^2 - Q_-^2 \rangle + f_2 \hat{Q}_+ \langle Q_+ \rangle + f_3 \langle Q_+^2 + Q_-^2 \rangle + f_4 \langle Q_+ \rangle^2 \right\} N, \quad (5)$$

with $Q_\pm = uQ^\dagger u \pm u^\dagger Q u^\dagger$ and $\hat{A} = A - \langle A \rangle / 2$ projects onto the off–diagonal elements of the operator A . Evidently, the charge matrices always have to appear quadratic since a virtual photon can never leave a diagram. The last two terms in eq.(5) are not observable since they lead to an equal em mass shift for the proton and the neutron, whereas the operator $\sim f_2$ to this order gives the em proton–neutron mass difference. In what follows, we will refrain from writing down such types of operators which only lead to an overall shift of masses or coupling constants. We note that in the pion and pion–nucleon sector, one can effectively count the electric charge as a small momentum or meson mass. This is based on the observation that $M_\pi / \Lambda_\chi \sim e / \sqrt{4\pi} = \sqrt{\alpha} \sim 1/10$ since $\Lambda_\chi \simeq 4\pi f_\pi = 1.2 \text{ GeV}$. It is thus possible to turn the dual expansion in small momenta/meson masses on one side and in the electric coupling e on the other side into an expansion with one generic small parameter. We also remark that from here we use the fine structure constant $\alpha = e^2/4\pi$ as the em expansion parameter.

We now turn to the two–nucleon sector, i.e. the four–fermion contact interactions without pion fields. Consider first the strong terms. Up to one derivative, the effective Lagrangian takes the form

$$\begin{aligned} \mathcal{L}_{NN}^{\text{str}} = & l_1 (N^\dagger N)^2 + l_2 (N^\dagger \vec{\sigma} N)^2 + l_3 (N^\dagger \langle \chi_+ \rangle N) (N^\dagger N) + l_4 (N^\dagger \hat{\chi}_+ N) (N^\dagger N) \\ & + l_5 (N^\dagger \vec{\sigma} \langle \chi_+ \rangle N) (N^\dagger \vec{\sigma} N) + l_6 (N^\dagger \vec{\sigma} \hat{\chi}_+ N) (N^\dagger \vec{\sigma} N) + \dots, \end{aligned} \quad (6)$$

where the ellipsis denotes terms with two (or more) derivatives acting on the nucleon fields. Similarly, one can construct the em terms. The ones without derivatives on the nucleon fields read

$$\begin{aligned} \mathcal{L}_{NN}^{\text{em}} = & N^\dagger \left\{ r_1 \langle Q_+^2 - Q_-^2 \rangle + r_2 \hat{Q}_+ \langle Q_+ \rangle \right\} N (N^\dagger N) \\ & + N^\dagger \vec{\sigma} \left\{ r_3 \langle Q_+^2 - Q_-^2 \rangle + r_4 \hat{Q}_+ \langle Q_+ \rangle \right\} N (N^\dagger \vec{\sigma} N) \\ & + N^\dagger \{ r_5 Q_+ + r_6 \langle Q_+ \rangle \} N (N^\dagger Q_+ N) + N^\dagger \vec{\sigma} \{ r_7 Q_+ + r_8 \langle Q_+ \rangle \} N (N^\dagger \vec{\sigma} Q_+ N) \\ & + r_9 (N^\dagger Q_+ N)^2 + r_{10} (N^\dagger \vec{\sigma} Q_+ N)^2. \end{aligned} \quad (7)$$

There are also various terms resulting from the insertion of the Pauli isospin matrices $\vec{\tau}$ in different $N^\dagger N$ binomials. Some of these can be eliminated by Fierz reordering, while the

others are of no importance for our considerations. Note that from now on we consider em effects. The leading CSB $\sim m_u - m_d$ has the same structure as the corresponding em term and thus its contribution can be effectively absorbed in the value of $E_0^{(2)}$, as defined below. We remark since Λ_{NN} is significantly smaller than Λ_χ , it does not pay to treat the expansion in the generic KSW momentum Q simultaneously with the one in the fine structure constant (as it is done e.g. in the pion–nucleon sector). Instead, one has to assign to each term a double expansion parameter $Q^n \alpha^m$, with n and m integers. Lowest order charge independence breaking is due to a term $\sim (N^\dagger \tau^3 N)^2$ whereas charge symmetry breaking at that order is given by a structure $\sim (N^\dagger \tau^3 N)(N^\dagger N)$. In the KSW approach, it is customary to project the Lagrangian terms on the pertinent NN partial waves. Denoting by β the 1S_0 partial wave for a given cms energy E_{cms} , the Born amplitudes for the lowest order CIB and CSB operators between the various two–nucleon states takes the form

$$\begin{aligned}\langle \beta, pp | \mathcal{L}_{NN}^{\text{em}} | \beta, pp \rangle &= - \left(\frac{Mp}{2\pi} \right) \alpha \left(E_0^{(1)} + E_0^{(2)} \right) , \\ \langle \beta, nn | \mathcal{L}_{NN}^{\text{em}} | \beta, nn \rangle &= - \left(\frac{Mp}{2\pi} \right) \alpha \left(E_0^{(1)} - E_0^{(2)} \right) ,\end{aligned}\tag{8}$$

where we will determine the coupling constant $E_0^{(1,2)}$ later on and also derive the scaling properties of the $E_{2n}^{(1,2)}$. The terms with the superscript ‘(1)’ refer to CIB whereas the second ones relevant for CSB are denoted by the superscript ‘(2)’. Higher order operators are denoted accordingly. There is, of course, also a CIB contribution to the np matrix element. To be consistent with the charge symmetric calculation of ref.[5], we absorb its effect in the constant D_2 , i.e. it amounts to a finite renormalization of D_2 and is thus not observable. In eq.(8), $p = \sqrt{ME_{\text{cms}}}$ is the nucleon cms momentum.

3. Consider now the effect of the charged to neutral pion mass difference δm^2 , see the upper left diagramm in fig. 1. OPE between two neutrons or two protons can obviously involve charged and neutral pions. The mass difference can be treated in two ways. As proposed in ref.[16], one can modify the pion propagator,

$$\Delta_\pi^{ab}(\ell) = \frac{i\delta^{ab}}{[\ell^2 - m_{\pi_0}^2 - \delta m^2 (1 - \delta^{3a})]} , \quad \delta m^2 = \frac{8\pi\alpha C}{f_\pi^2} ,\tag{9}$$

with ℓ the pion four–momentum and ‘ a, b ’ isospin indices. Since we are interested only in the leading corrections $\sim \delta m^2 \sim \alpha$, it suffices to work with the expanded form of eq.(9),

$$\Delta_\pi^{ab}(\ell) = \frac{i\delta^{ab}}{\ell^2 - m_{\pi_0}^2} + \delta m^2 \frac{i\delta^{ab}(1 - \delta^{3a})}{(\ell^2 - m_{\pi_0}^2)^2} + \mathcal{O}((\delta m^2)^2) .\tag{10}$$

From this we conclude that OPE diagrams with different pion masses have the isospin structure $O_{12} = \tau_{(1)a} \Delta_\pi^{ab} \tau_{(1)b} = (a + b) \vec{\tau}_{(1)} \cdot \vec{\tau}_{(2)} - b \tau_{(1)}^3 \tau_{(2)}^3$ and lead to CIB since

$$\begin{aligned}\langle pp | O_{12} | pp \rangle_{\text{Cs}} &= \langle nn | O_{12} | nn \rangle = \frac{a}{4} , \\ \langle np | O_{12} | np \rangle + \langle np | O_{12} | pn \rangle &= \frac{a}{4} + \frac{b}{2} ,\end{aligned}\tag{11}$$

for the various isospin components of the two-nucleon system and 'Cs' stands for Coulomb-subtracted. Obviously, these effects are of order αQ^{-2} . The np amplitude was already calculated by KSW. We have worked out the leading corrections $\Delta\mathcal{A} = \mathcal{A}_{nn} - \mathcal{A}_{np} = \mathcal{A}_{pp}^{\text{Cs}} - \mathcal{A}_{np}$ due to the pion mass difference. The pertinent diagrams are shown in fig. 1. We follow the notation of KSW in that we call these corresponding three amplitudes $\mathcal{A}_{1,-2}^{II,III,IV}$ where the first (second) subscript refers to the power in α (Q) and the superscripts to the first three diagrams of the figure. We find

$$\begin{aligned}\Delta\mathcal{A}_{1,-2}^{II} &= \Gamma \left[\frac{1}{4p^2} \ln \left(1 + \frac{4p^2}{m^2} \right) - \frac{1}{m^2 + 4p^2} \right] , \\ \Delta\mathcal{A}_{1,-2}^{III} &= \Gamma \left(\frac{mM\mathcal{A}_{-1}}{4\pi} \right) \left[\frac{1}{pm} \arctan \frac{2p}{m} + \frac{i}{2pm} \ln \left(1 + \frac{4p^2}{m^2} \right) - \frac{1 + \frac{2ip}{m}}{m^2 + 4p^2} \right] \\ \Delta\mathcal{A}_{1,-2}^{IV} &= \Gamma \left(\frac{mM\mathcal{A}_{-1}}{4\pi} \right)^2 \left[\frac{i}{m^2} \arctan \frac{2p}{m} - \frac{1}{2m^2} \ln \left(\frac{m^2 + 4p^2}{\mu^2} \right) + \frac{1}{m^2} - \frac{1}{2} \frac{1 + \frac{2ip}{m}}{m^2 + 4p^2} \right] \\ \Gamma &= -\delta m^2 \frac{g_A^2}{2f_\pi^2} , \quad \delta m^2 = m_{\pi^\pm}^2 - m_{\pi^0}^2 , m^2 = m_{\pi^0}^2 + 2\delta m^2 ,\end{aligned}\tag{12}$$

with $\mathcal{A}_{-1} \equiv \mathcal{A}_{0,-1}$ the leading term in the expansion of the np 1S_0 amplitude [5]

$$\mathcal{A}_{-1} = -\frac{C_0(\mu)}{1 + C_0(\mu)M(\mu + ip)/4\pi} .\tag{13}$$

Here, μ is the PDS regularization scale. Note while the diagrams II and III are convergent, IV diverges logarithmically. Therefore, the Lagrangian must contain a counterterm of the structure $E_0^{(1)}(\mu)\alpha(N^\dagger\tau^3N)(N^\dagger\tau^3N)$ (cf. section 2) since it is needed to make the amplitude scale-independent. Note that for graph IV we have used the same subtraction as performed in [5]. Consequently, for operators of this type with $2n$ derivatives we can establish the scaling property $E_{2n}^{(1)} \sim Q^{-2+n}$. This does not contradict the KSW power counting for the isospin symmetric theory since $\alpha \ll 1$. Stated differently, the leading CIB term of order αQ^{-2} is numerically much smaller than the strong leading order contribution $\sim Q^{-1}$. The insertion from this contact term is shown in the last diagram of fig. 1 and leads to an additional contribution to $\Delta\mathcal{A}$. In complete analogy, we can treat the leading order CSB effect which is due to an operator of the form $\alpha E_0^{(2)}(N^\dagger\tau_3N)(N^\dagger N)$. This term is, however, finite. Putting pieces together, we get

$$\Delta\mathcal{A}_{1,-2,\text{pp}}^{VI} = -\alpha \left(E_0^{(1)} + E_0^{(2)} \right) \left[\frac{\mathcal{A}_{-1}}{C_0} \right]^2 , \quad \Delta\mathcal{A}_{1,-2,\text{nn}}^{VI} = -\alpha \left(E_0^{(1)} - E_0^{(2)} \right) \left[\frac{\mathcal{A}_{-1}}{C_0} \right]^2 ,\tag{14}$$

where the coupling constants $E_0^{(1,2)}(\mu)$ obeys the renormalization group equations,

$$\begin{aligned}\mu \alpha \frac{dE_0^{(1)}(\mu)}{d\mu} &= \alpha \frac{M}{2\pi} E_0^{(1)}(\mu) C_0(\mu) \mu - \delta m^2 \frac{M^2 g_A^2}{32\pi^2 f_\pi^2} C_0^2(\mu) , \\ \mu \alpha \frac{dE_0^{(2)}(\mu)}{d\mu} &= \alpha \frac{M}{2\pi} E_0^{(2)}(\mu) C_0(\mu) \mu .\end{aligned}\tag{15}$$

Note that from here on we do no longer exhibit the scale dependence of the various couplings constants $E_0^{(1,2)}, C_{0,2}, D_2$. We can now relate the pp and nn scattering lengths to the np one (of course, in the pp system Coulomb subtraction is assumed),

$$\begin{aligned}\frac{1}{a_{pp}} &= \frac{1}{a_{np}} - \frac{4\pi\alpha(E_0^{(1)} + E_0^{(2)})}{MC_0^2} + \Delta, \\ \frac{1}{a_{nn}} &= \frac{1}{a_{np}} - \frac{4\pi\alpha(E_0^{(1)} - E_0^{(2)})}{MC_0^2} + \Delta, \\ \Delta &= \delta m^2 \frac{g_A^2(-C_0M(m - 2\mu) + 8\pi + C_0Mm \ln(m^2/\mu^2))}{16\pi m C_0 f_\pi^2}.\end{aligned}\tag{16}$$

For the effective ranges, we have only CIB

$$r_{nn} = r_{pp} = r_{np} + \delta m^2 \frac{g_A^2(C_0M\mu + 4\pi)(C_0M(3\mu - 2m) + 12\pi)}{6\pi M m^4 C_0^2 f_\pi^2}.\tag{17}$$

Note that this last relation is scale-independent and that it does not contain any unknown parameter. We remark that for the CIB scattering lengths difference the pion contribution alone is *not* scale-independent and can thus never be uniquely disentangled from the contact term contribution $\sim E_0^{(1)}$. While the leading OPE contribution resembles the result obtained in meson exchange models, the mandatory appearance of this contact term is a distinctively new feature of the effective field theory approach. It is easy to classify the leading and next-to-leading em corrections to these results. At order αQ^{-1} , one has the contribution from two potential pions with the pion mass difference and also contact interactions with two derivatives. Effects due to the charge dependence of the pion-nucleon coupling constants, i.e. isospin breaking terms from $\mathcal{L}_{\pi N}^{\text{em}}$, only start to contribute at order αQ^0 . Such effects are therefore suppressed by two orders of Q compared to the leading terms. This finding is in agreement with the various numerical analyses performed in potential models. We now turn to CSB. Here, to leading order there is simply a four-nucleon contact term proportional to the constant $E_0^{(2)}$. Its value can be determined from a fit to the empirical value given in eq.(2). First corrections to the leading order CSB effect are classified below.

4. We now turn to the numerical analysis considering exclusively the 1S_0 channel. At leading order, all parameters can be fixed from the pertinent scattering length. Already in ref.[5] it was pointed out that there are various possibilities of fixing the next-to-leading order constants. One is to stay at low energies (i.e. fitting a and r) or performing an overall fit to the phase shift up to momenta of about 200 MeV. In this latter case, however, the resulting low energy parameters deviate from their empirical values and also, at $p \sim 150$ MeV, the correction is as big as the leading term. Since we are interested in small effects like CIB and CSB, we stick to the former approach and use the 1S_0 np phase shift around $p = 0$ to fix the parameters as shown by the dashed curve in fig.2. More precisely, we fit the parameters $C_{0,2}, D_2$ to the np phase shift under the condition that the scattering length and effective range are exactly reproduced. The two new parameters $E_0^{(1,2)}$ are determined from the nn and pp scattering lengths. The resulting parameters at

$\mu = m$ are of natural size,

$$\begin{aligned} C_0 &= -3.46 \text{ fm}^2, & C_2 &= 2.75 \text{ fm}^4, & D_2 &= 0.07 \text{ fm}^4, \\ E_0^{(1)} &= -6.47 \text{ fm}^2, & E_0^{(2)} &= 1.10 \text{ fm}^2. \end{aligned} \quad (18)$$

To arrive at the curves shown in fig.2, we have used the physical masses for the proton and the neutron. We stress again that the effect of the em terms $\sim E_0^{(1,2)}$ is small because of the explicit factor of α not shown in eq.(18). Having fixed these parameters, we can now predict the nn 1S_0 phase shift as depicted by the solid line in fig.2. It agrees with nn phase shift extracted from the Argonne V18 potential (with the scattering length and effective range exactly reproduced) up to momenta of about 100 MeV. The analogous curve for the pp system (not shown in the figure) is close to the solid line since the CSB effects are very small.

5. Finally, we can give a summary of the various leading (LO), next-to-leading (NLO) and next-to-next-to-leading order (NNLO) contributions to CIB and CSB, with respect to the expansion in Q , to leading order in α and the light quark mass difference. Consider first CIB.

LO, αQ^{-2} Electromagnetic corrections to one-pion exchange (pion mass difference) and electromagnetic four-nucleon contact interactions with no derivatives.

NLO, $\alpha Q^{-1}, \varepsilon Q^{-1}$ Em corrections to two-pion exchange, four-nucleon contact terms with two derivatives and insertions proportional to the strong neutron-proton mass splitting.

NNLO, αQ^0 Em corrections to three-pion exchange and four-nucleon contact terms with four derivatives as well as charge dependent coupling constants from the electromagnetic pion-nucleon Lagrangian.

Note that there are no CIB effects due to the light quark mass difference linear in $\varepsilon = m_u - m_d$,^{#7} except the trivial effects related to the strong pn mass difference. We now turn to CSB. The pattern of the various contributions looks different:

LO, $\alpha Q^{-2}, \varepsilon Q^{-2}$ Electromagnetic and strong isospin-breaking four-nucleon contact interactions with no derivatives.

NLO, $\alpha Q^{-1}, \varepsilon Q^{-1}$ Em four-nucleon contact terms with two derivatives, strong isospin-breaking contact interaction with two derivatives and insertions proportional to the neutron-proton mass difference.

Here, we assume the same scaling in powers of Q for the em and the strong coupling constants. We remark that the leading order CSB effects do not modify the effective range but the corresponding scattering length.

^{#7}One could also use the dimensionless small parameter $\epsilon = (m_u - m_d)/(m_u + m_d) \simeq 1/3$, see e.g. ref.[8]. We prefer to keep the notation which is most commonly used in the pion-nucleon sector, see e.g. ref.[21].

6. In summary, we have considered electromagnetic and strong isospin violation in low-energy nucleon–nucleon scattering in the effective field theory formalism developed in ref.[5]. In particular, the leading charge independence breaking effect is due to a combination of the neutral to charged pion mass difference in one-pion exchange diagrams together with an electromagnetic four-nucleon contact term. Its corresponding coupling constant scales as Q^{-2} but is numerically suppressed by the explicit appearance of the fine structure constant $\alpha \sim 1/137$. We have shown how the KSW power counting has to be modified in the presence of isospin violating operators. We explicitly evaluated the 1S_0 phase shifts for the np , nn and Coulomb-subtracted pp systems at next-to-leading order. In addition, we have given a general classification of the various CIB and CSB corrections. It would be interesting to extend this formalism to other partial waves and to higher energies so as to investigate e.g. isospin violation in pion production.

Acknowledgments

We thank Vincent Stoks for providing us with the nn phase shift.

References

- [1] G. Miller, B.M.K. Nefkens and I. Slaus, Phys. Rep. 194 (1990) 1.
- [2] S. Weinberg, Phys. Lett. B295 (1992) 114.
- [3] S.R. Beane, V. Bernard, T.-S.H. Lee, Ulf-G. Meißner and U. van Kolck, Nucl. Phys. A618 (1997) 381.
- [4] J. Bergstrom et al., Phys. Rev. C57 (1998) 3202.
- [5] D.B. Kaplan, M.J. Savage and M.B. Wise, Phys. Lett. B424 (1999) 329; Nucl. Phys. B534 (1998) 329.
- [6] T. Mehen and I.W. Stewart, nucl-th/9809095.
- [7] D.B. Kaplan, nucl-th/9901003.
- [8] U. van Kolck, Ph.D. Thesis, University of Texas, Austin (1993).
- [9] U. van Kolck, Phys. Rev. C49 (1994) 2932.
- [10] U. van Kolck, M.C.M. Rentmeester, J.L. Friar, T. Goldman and J.J. de Swart, Phys. Rev. Lett. 80 (1998) 4386.
- [11] X. Kong and F. Ravndal, nucl-th/9811076.
- [12] B.R. Holstein, nucl-th/9901041.
- [13] G. Ecker, J. Gasser, A. Pich and E. de Rafael, Nucl. Phys. B321 (1989) 311.
- [14] R. Urech, Nucl. Phys. B433 (1995) 234.
- [15] H. Neufeld and H. Rupertsberger, Z. Phys. C68 (1995) 91.
- [16] Ulf-G. Meißner, G. Müller and S. Steininger, Phys. Lett. B406 (1997) 154, (E) *ibid* B407 (1997) 434.
- [17] M. Knecht and R. Urech, Nucl. Phys. B519 (1998) 329.

- [18] Ulf-G. Meißner and S. Steininger, Phys. Lett. B419 (1998) 403.
- [19] G. Müller and Ulf-G. Meißner, in preparation.
- [20] N. Fettes, Ulf-G. Meißner and S. Steininger, Nucl. Phys. A640 (1998) 199.
- [21] V. Bernard, N. Kaiser and Ulf-G. Meißner, Int. J. Mod. Phys. E4 (1995) 193.
- [22] V.G.J. Stoks et al., Phys. Rev. C49 (1994) 2950.

Figures

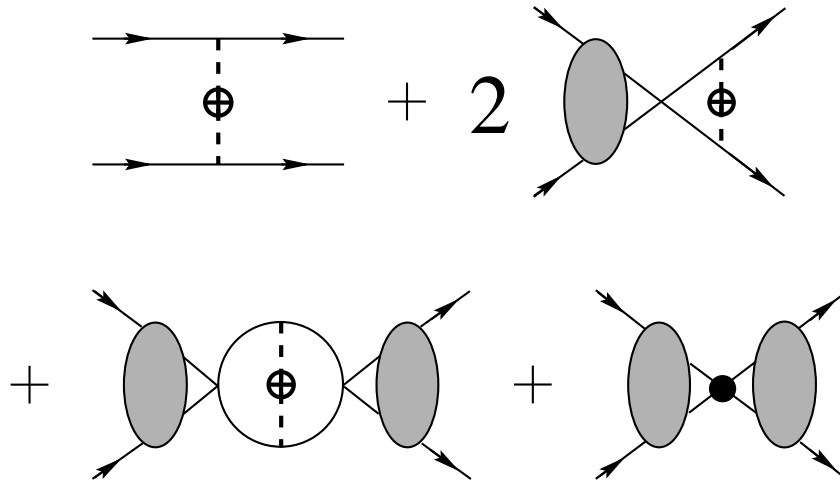


Figure 1: Relevant graphs contributing to charge independence breaking at leading order αQ^{-2} . The blob stands for the resummation of the lowest order $(N^\dagger N)^2$ contact terms $\sim C_0$. The open (filled) circle denotes a pion mass insertion $\sim \delta m^2$ (an insertion of the leading four-nucleon operators $\sim \alpha E_0^{(1)}$). For charge symmetry breaking, the leading contribution is given by the last diagram with the filled circle denoting an insertion $\sim \alpha E_0^{(2)}$.

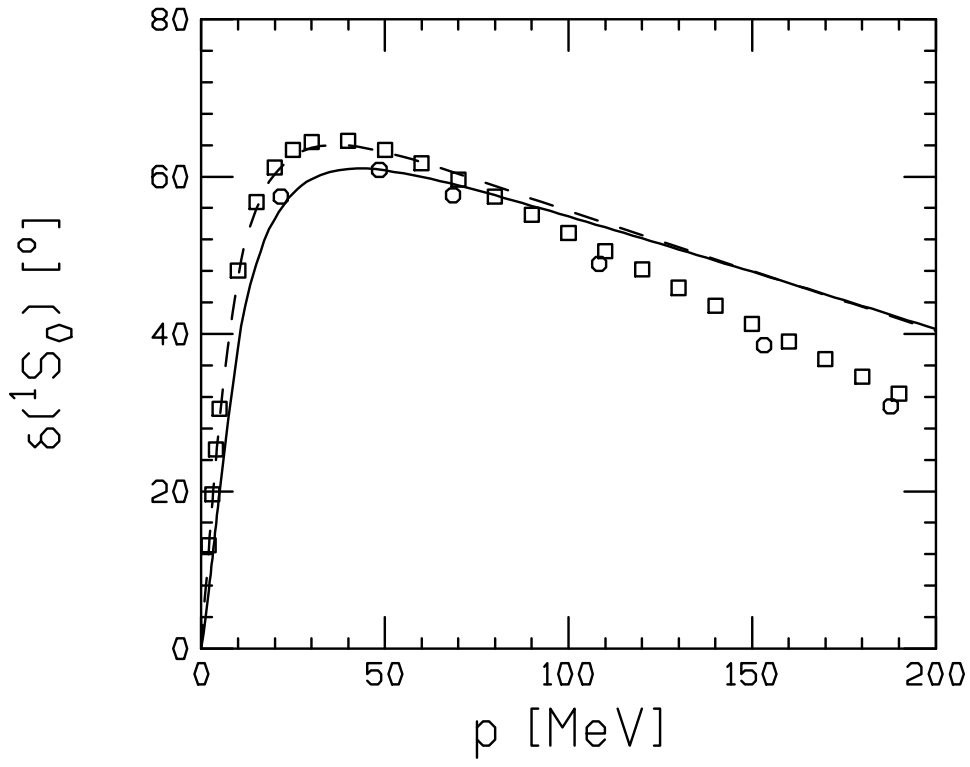


Figure 2: 1S_0 phase shifts for the np (dashed line) and nn (solid line) systems versus the nucleon cms momentum. The empirical values for the np case (open squares) are taken from the Nijmegen analysis [22]. The open octagons are the nn “data” based on the Argonne V18 potential.